

Constraining Superfluidity in Dense Matter from the Cooling of Isolated Neutron Stars

Spencer Beloin¹, Sophia Han¹, Andrew W. Steiner^{1,2}, and Dany Page³

¹*Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996, USA*

²*Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA and*

³*Instituto de Astronomía, Universidad Nacional Autónoma de México, Mexico D.F. 04510, Mexico*

We present a quantitative analysis of superfluidity and superconductivity in dense matter from observations of isolated neutron stars in the context of the minimal cooling model. Our new approach produces the best fit neutron triplet superfluid critical temperature, the best fit proton singlet superconducting critical temperature, and their associated statistical uncertainties. We find that the neutron triplet critical temperature is $(1.36 \pm 0.11) \times 10^8$ K and that the proton singlet critical temperature is $7.1^{+1.6}_{-1.3} \times 10^9$ K. However, we also show that this result only holds if the Vela neutron star is not included in the data set. If Vela is included, the gaps increase significantly in order to attempt to reproduce Vela's lower temperature given its young age. Further including neutron stars believed to have carbon atmospheres does not change the parameter estimates significantly. Our method demonstrates that continued observations of isolated neutron stars can quantitatively constrain the nature of superfluidity in dense matter.

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I. INTRODUCTION

Neutron stars, the remnants of the gravitational collapse of ~ 8 to $20 M_\odot$ main-sequence stars, contain matter with densities at least several times larger than the densities at the center of atomic nuclei [1]. Matter at these densities is difficult to probe in the laboratory, except at high temperatures, which confounds the extraction of dense matter properties from experiment. Thus, neutron stars are a unique laboratory for the study of dense and strongly-interacting matter.

Current constraints from neutron star mass and radius observations determine the equations of state of dense matter (EOS) above the nuclear saturation densities to within about a factor of two (see recent constraints in Refs. [2, 3] or an alternate perspective in Ref. [4]). Recent progress in nuclear theory constrains the energy per baryon of neutron matter at the saturation density to within a few MeV [5]. However, the EOS alone is not enough to fully describe dense matter. Almost all neutron star observables also require some knowledge of how energy and momentum are transported in dense matter. Transport properties, in turn, are strongly affected by the presence of superconductivity and superfluidity [6].

At the end of a supernova, the neutron star is born with a core temperature $\sim 10^{11}$ K, and, in some cases, a measurable velocity with respect to the remnant. Except for a thin shell at the surface, the neutron star becomes isothermal after a few hundred years. In isolated neutron stars without a companion, the temperature decreases (unless heated by magnetic field dissipation or some dark matter-related process) at a rate determined by the nature of dense matter [7–9]. In the first 10^5 years, cooling is dominated by the emission of neutrinos from the core. The cooling rate depends on the temperature dependence of the neutrino luminosity and the specific heat of dense matter. Afterwards, cooling is dominated

by photon emission. If one obtains temperature and age estimates from a number of cooling neutron stars, the comparison of theoretical models to data results in a constraint on the nature of superfluidity in dense matter.

II. METHOD

There are several isolated neutron stars where age estimates are available and where X-ray data provides an estimate of the surface temperature. The extraction of the surface temperature, however, depends on the composition of the atmosphere. Older neutron stars are expected to have atmospheres made of iron-peak elements and these atmospheres are well fit by black body models giving black body radii in the range of 10–13 km expected from theoretical models [10]. The inferred radii from black body fits to younger stars are often much smaller than expected, leading to the idea that younger isolated neutron stars may have light-element atmospheres. In most objects, only black body and H atmosphere fits to X-ray data are available. The temperature profile of the star depends on the composition of the envelope, the region between the photosphere and a boundary density near 10^{10} g/cm² defined so that the luminosity in the envelope is equal to the total luminosity of the star. In the case of a light-element atmosphere, the presence of light elements in the envelope can modify the inferred surface temperature. Light-element envelopes are not expected with iron-peak atmospheres described by black body models, as light elements in the envelope will inevitably make their way to the surface.

Similar to Ref. [7], we use the temperature implied by H atmosphere models for younger stars (typically less than about 10^5 years) in which black body radii are too small to be realistic. In these cases, we also vary the amount of light-elements in the envelope. In older

stars, black body radii are realistic. This result is consistent with the notion that neutron star atmospheres evolve from light elements to iron-peak elements over time through nuclear fusion. In some cases, hydrogen atmosphere models imply relatively large magnetic fields, as large as 10^{12} G, which may strongly modify the X-ray spectrum. We ignore the potential impact of magnetic fields and leave their consideration to future work. There are a few objects for which neither H nor black body atmospheres imply a realistic neutron star radius, but where carbon atmospheres fit well. This is the case for the neutron star located in Cassiopeia A and XMMU J1732 located in HESS J1731–347, which we include in our analysis along with the possibility that they also may contain light elements in their envelopes.

Pulsar ages can be estimated from the spin-down timescale, $t_{\text{sd}} = P/(2\dot{P})$, an age estimate assuming an evolution with a dipolar magnetic field. In many cases, the uncertainty in the spin-down age is not known, and a fiducial factor of 3 uncertainty is assumed. Alternatively, if a neutron star can be associated with a nearby supernova remnant and its proper motion can be measured, one can determine the kinetic age, t_{kin} . We presume kinetic ages are more accurate than spin-down ages, but this is not certain. The observational data set is summarized in Table I.

We employ the minimal cooling model from Ref. [7], assuming that the neutron star is made entirely of neutrons, protons, and leptons, and that the direct Urca process does not occur. This model takes advantage of the fact that the bulk thermodynamics of the equations of state and the neutron star mass do not to significantly affect the cooling. We assume the Akmal-Pandharipande-Ravenhall (APR; Ref. [44]) equations of state and also that all isolated neutron stars have a mass of $1.4 M_{\odot}$. We assume no additional cooling processes occur due to the presence of deconfined quarks, Bose condensates or exotic (i.e. heavy) hadrons. The simplification provided by the minimal cooling model is important because it allows us to decrease our parameter space which is already relatively large (as described below). We also ignore any possible effects on the cooling from magnetic fields or rotation.

In the minimal model, the principal unknown quantities in dense matter which impact neutron star cooling are the neutron superfluid and proton superconducting gaps. Superfluidity and superconductivity exponentially suppress the specific heat and modify the neutrino emissivities (for a review see Ref. [6]). These effects begin when the temperature becomes lower than the critical temperature. In the original BCS theory of superconductivity, the critical temperature and the value of the gap at zero temperature are related by $\Delta(T = 0) \simeq 1.8 k_B T_c$. The BCS approximation to superconductivity does not necessarily apply in the strongly-interacting nucleon fluid, but we retain the standard practice of assuming that the BCS relation is approximately correct.

Neutron superfluidity in the singlet (1S_0) channel is

present in the neutron star crust, but the critical temperatures are too large to be constrained by the data of neutron stars older than a few hundred years. Proton singlet superconductivity in the outer core and neutron 3P_2 superfluidity in the inner core, on the other hand, are the most important parameters in the minimal cooling model and can be constrained by neutron stars with the ages found in our data set. Superfluid gaps suppress heat capacity for temperatures well below T_c (but increase heat capacity at temperature just below T_c). Superfluidity and superconductivity also allow a new neutrino emission process induced by the formation of Cooper pairs. This cooling process is included, along with the correction due to suppression in the vector channel [45–48].

Theoretical calculations of the neutron and proton critical temperatures in the neutron star core appear approximately as Gaussian functions of the Fermi momentum [6]. Pairing is suppressed at low densities as the interparticle spacing is increased, and also suppressed at high densities as the repulsion between nucleons quenches the attractive interaction. In this work, we assume that both the proton singlet and neutron triplet critical temperatures can be described by the Gaussian form

$$T_c(k_F) = T_{c,\text{peak}} \exp \left[\frac{(k_F - k_{F,\text{peak}})^2}{2\Delta k_F^2} \right] \quad (1)$$

with parameters $T_{c,\text{peak}}$, $k_{F,\text{peak}}$ and Δk_F .

In order to avoid overcounting models where the gaps vanish, we constrain $k_{F,p,\text{peak}}$ and $k_{F,n,\text{peak}}$ to lie between the crust-core transition (taken to be at $n_B = 0.09 \text{ fm}^{-3}$) and the central density of a $1.4 M_{\odot}$ neutron star (at $n_B = 0.545 \text{ fm}^{-3}$). This implies $0.481 \text{ fm}^{-3} < k_{F,p,\text{peak}} < 1.304 \text{ fm}^{-3}$ and $1.418 \text{ fm}^{-3} < k_{F,n,\text{peak}} < 2.300 \text{ fm}^{-3}$. To avoid overcounting models where the gap is nearly independent of density, we also enforce $\Delta k_{F,p} < 1.304 \text{ fm}^{-3}$ and $\Delta k_{F,n} < 2.300 \text{ fm}^{-3}$. Finally, we constrain our critical temperatures to be smaller than 10^{10} K since the fit is insensitive to larger values.

In the case where one is fitting a model to data with small uncertainties in the independent variable, the χ^2 procedure gives a unambiguous procedure to determine the best fit assuming that the data points are statistically independent and have a normally distributed uncertainty in the dependent variable. When the data points are presented in a two-dimensional plane with comparable uncertainties in both axes, there is no unique “correct” fitting procedure (this conclusion holds in both the frequentist and Bayesian paradigms). In the frequentist picture, the lack of a unique fitting procedure has led to the use of several methods including orthogonal least squares, orthogonal regression, reduced major axis regression [49]. Several of these procedures are often referred to by different names. Reduced major axis regression is also referred to as geometric mean regression in Ref. [50] and a linear version obtaining the so-called “impartial line” was first used in Ref. [51]. This ambiguity is part of the reason why more quantitative fits to neutron star cooling data

Star	$\log_{10}(t_{\text{sd}}/\text{yr})$ or $\log_{10}(t_{\text{kin}}/\text{yr})$	B (G)	$\log_{10}(T/\text{K})$	atmosphere model	mass	radius	Ref.
Cas A NS	2.52 (observed)	8×10^{10}	$6.26^{+0.02}_{-0.02}$	C	$1.4 M_{\odot}^*$	12-15 km	[11]
			$6.447^{+0.012}_{-0.012}$	H	$1.4 M_{\odot}^*$	4 km	[11]
			$6.653^{+0.007}_{-0.007}$	BB	$1.4 M_{\odot}^*$	< 1 km	[11]
PSR J1119–6127	3.20 (sd)	4.1×10^{13}	$6.09^{+0.08}_{-0.08}$	HA	$1.4 M_{\odot}^*$	10 km*	[12]
			$6.38^{+0.02}_{-0.02}$	BB	$1.4 M_{\odot}^*$	2.7 ± 0.7 km	[12]
RX J0822–4247	$3.57^{+0.04}_{-0.04}$ (kin)	$\sim 10^{12}$ G	$6.24^{+0.04}_{-0.04}$	HA	$1.4 M_{\odot}^*$	10 km*	[13]
			$6.65^{+0.04}_{-0.04}$	BB	$1.4 M_{\odot}^*$	≈ 2 km	[13]
1E 1207.4–5209 ⁺⁺	$3.85^{+0.48}_{-0.48}$ (kin; [14])	3×10^{12} G	$6.21^{+0.07}_{-0.07}$	HA	$1.4 M_{\odot}^*$	10 km*	[15]
			$6.48^{+0.01}_{-0.01}$	BB	$1.4 M_{\odot}^*$	< 1.5 km	[16, 17]
PSR J1357–6429	3.86 (sd)	8×10^{12} G	$5.88^{+0.04}_{-0.04}$	HA	$1.5 - 1.6 M_{\odot}$	10 km*	[18]
			$6.23^{+0.05}_{-0.05}$	BB	$1.5 - 1.6 M_{\odot}$	2.5 ± 0.5 km	[18]
RX J0002+6246 [†]	$3.96^{+0.08}_{-0.08}$ (kin)		$6.03^{+0.03}_{-0.03}$	HA			[19]
			$6.15^{+0.11}_{-0.11}$	BB			[19]
PSR B0833–45	3.97 ± 0.23 (kin; [20])	3×10^{12} G	$5.83^{+0.02}_{-0.02}$	HA	$1.4 M_{\odot}^*$	13 km	[21]
			$6.18^{+0.02}_{-0.02}$	BB	$1.4 M_{\odot}^*$	2.1 ± 0.2 km	[21]
PSR B1706–44	4.24 (sd; [22])	3×10^{12} G	$5.80^{+0.13}_{-0.13}$	HA	$1.45 - 1.59 M_{\odot}$	13 km	[23]
			$6.22^{+0.04}_{-0.04}$	BB	$1.4 M_{\odot}^*$	< 6 km	[22]
XMMU J1732–344	$4.43^{+0.17}_{-0.43}$ (kin; [24])	$\sim 10^{10-11}$ G	$6.25^{+0.01}_{-0.0045}$	C			[25]
PSR J0538+2817	$4.47^{+0.05}_{-0.06}$ (sd; [26])	$\sim 10^{12}$ G	$6.05^{+0.1}_{-0.1}$	HA	$1.4 M_{\odot}^*$	10.5 km	[27]
			$6.327^{+0.007}_{-0.007}$	BB	$1.4 M_{\odot}^*$	< 2 km	[28]
PSR B2334+61	4.61 (sd)	10^{10-12} G	$5.74^{+0.13}_{-0.13}$	HA	$1.4 M_{\odot}^*$	10-13 km	[29]
			$6.10^{+0.15}_{-0.15}$	BB	$1.4 M_{\odot}^*$	< 2 km	[29]
PSR B0656+14	5.04 (sd; [30])	5×10^{12} G [31]	$5.71^{+0.03}_{-0.04}$	BB	$1.4 M_{\odot}^*$	12-17 km	[32]
PSR B0633+1748	5.53 (sd)		$5.75^{+0.04}_{-0.05}$	BB	$1.4 M_{\odot}^*$	10 km*	[33]
RX J1856.4–3754 [‡]	$5.70^{+0.05}_{-0.25}$ (kin; [34])	4×10^{12} G	$5.75^{+0.15}_{-0.15}$	BB		14 km	[35, 36]
PSR B1055–52 [§]	5.73 (sd)	4×10^{12} G	$5.88^{+0.08}_{-0.08}$	BB	$1.4 M_{\odot}^*$	13 km	[37]
PSR J2043+2740 [¶]	6.08 (sd)	10^{14} G	$5.64^{+0.08}_{-0.08}$	HA		10 km*	[30]
PSR J0720.4-3125	6.11 (sd; [38])	10^{13} G [39]	$5.75^{+0.20}_{-0.20}$	BB	$1.4 M_{\odot}^*$	11-13 km	[40]

TABLE I: The data set used in the current work, adapted from the earlier work in Refs. [7] and [41]. As in Ref. [7] we favor kinetic ages over spin-down ages where possible. References are given in column 2 only where our ages differ from the values used in Ref. [7]. We use H atmosphere (HA) fits to stars less than 10^5 years and blackbody (BB) fits for older stars. In some of the H atmosphere fits, a magnetic field was used (either as a fixed value or as a fit parameter), and this is indicated in the fourth column (mHA). Notes: (*) This value was assumed not derived. (||) For the hydrogen atmosphere fit, we use the redshifted temperature from Ref. [27], $10^{6.04}$, instead of the value reported as $10^{5.94}$ in Ref. [41]. (‡) As in Ref. [7], we use a range determined by the colder blackbody component from Ref. [35] and the warmer blackbody component in Ref. [36]. (§) We have used the updated information from Ref. [37] as in Ref. [41] over the values in Ref. [7]. (¶) We use a hydrogen atmosphere fit for this source since a blackbody fit is not available. (5) As in Ref. [7] we use a range determined by the cold and warm components from the blackbody model in Ref. [40]. (†) Ref. [42] claims this is not a neutron star. (++) As discussed in Ref. [7], Ref. [43] suggests that this star may be accreting due to its spin-down behavior.

have not yet been performed before this work.

We choose to proceed using Bayesian inference, with

$$P(M|D) \propto P(D|M)P(M) \quad (2)$$

where $P(M)$ is the prior distribution for the model M (which in our case has six parameters for superfluidity/superconductivity and one parameter for the envelope composition of each star) and $P(D|M)$ is the likelihood function obtained from the neutron star cooling data. The quantity $P(M|D)$ is the probability distribution that we want to obtain, the probability of the theoretical model given the data. In the Bayesian picture, the

non-uniqueness in the fitting procedure described above is manifest in the undetermined prior distribution which one must choose in order to proceed.

Because the uncertainties in the neutron star cooling data are often presented in terms of the logarithms of temperature and time, we choose to write the likelihood in terms of new variables \hat{t} and \hat{T} ,

$$\begin{aligned} \hat{t} &\equiv \frac{1}{5} \log_{10} \left(\frac{t}{10^2 \text{ yr}} \right) \quad \text{and} \\ \hat{T} &\equiv \frac{1}{2} \log_{10} \left(\frac{T}{10^5 \text{ K}} \right) \end{aligned} \quad (3)$$

which are defined so that typical values are between 0 and 1.

We assume that our data set is Gaussian in both variables \hat{t} and \hat{T} , and can thus be specified as \hat{t}_j , \hat{T}_j , $\delta\hat{t}_j$, and $\delta\hat{T}_j$. (Our uncertainties are sufficiently small that the distinction between normal and log-normal distributions will not strongly impact our qualitative results.) The composition of the envelope is parameterized by a quantity η which takes values from 0 to 10^{-7} , larger values representing a larger contribution from light elements [52]. The cooling code computes three different cooling curves, for $\eta = 0$, $\eta = 10^{-12}$, and for $\eta = 10^{-7}$ and results for other values of η are obtained through linear interpolation. For neutron stars which have hydrogen atmospheres, the likelihood function is

$$\mathcal{L}_H \propto \prod_j \int d\hat{t} \sqrt{\left\{ \left[\frac{d\hat{T}(\eta_j, \hat{t})}{d\hat{t}} \right]^2 + 1 \right\}} \exp \left\{ \frac{-(\hat{t} - \hat{t}_j)^2}{2(\delta\hat{t}_j)^2} \right\} \times \exp \left\{ \frac{-(\hat{T}(\eta_j, \hat{t}) - \hat{T}_j)^2}{2(\delta\hat{T}_j)^2} \right\}. \quad (4)$$

The overall normalization is unspecified and is not necessary for our results. The full likelihood is $\mathcal{L} = \mathcal{L}_H \mathcal{L}_C \mathcal{L}_{bb}$, a product over a similar expression for the carbon stars (when they are included in the analysis) and a similar expression for the black body stars (for which $\eta = 0$). Note that this likelihood function reduces exactly to the likelihood function for the traditional χ^2 procedure in the limiting cases that one of the two variables has a small uncertainty. The square root operates as a line element, specifying how one defines a distance when integrating the cooling curve along the data. The ambiguity in defining this distance is the exact same as the choice in using different frequentist regression techniques. Our approach makes this ambiguity explicit.

This technique is very similar to the recent determination of the mass-radius curve given neutron star mass and radius observations (the formalism was first developed in Ref. [53] and most recently updated in Ref. [10]). There are two significant differences. First, the term under the square root was ignored, appropriate since the radius depends only very weakly with the neutron star mass. Second, the data in that case is not Gaussian in either mass or radius so a more complicated probability distribution was used rather than the product of two Gaussians employed in this work.

In practice, the cooling curves are specified as arrays, $\hat{T}(\eta, \hat{t}) \rightarrow [\hat{T}_k(\eta), \hat{t}_k]$ and finite differencing gives the derivative $[d\hat{T}(\eta)/d\hat{t}]_k$. To a good approximation we can

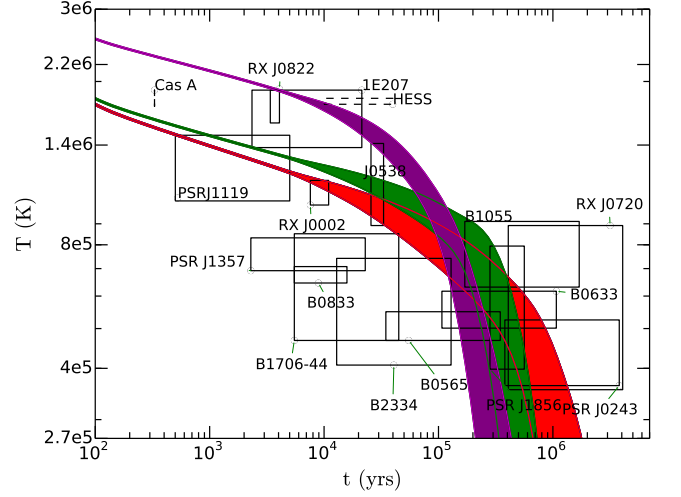


FIG. 1: Neutron star cooling data and three bands which show the $\pm 1\sigma$ uncertainties on the cooling curves (due to superfluidity and superconductivity) having removed both Vela and the carbon stars from the fit (even though they are included in the plot for comparison). The three bands represent three different values of η , 10^{-7} , 10^{-12} and 10^{-17} . These results correspond to the parameter limits in the second column of Table II.

replace the integral by a sum

$$\mathcal{L}_H \propto \prod_j \sum_k \sqrt{\left\{ \left[\frac{d\hat{T}(\eta_j)}{d\hat{t}} \right]_k^2 + 1 \right\}} \exp \left\{ \frac{-(\hat{t}_k - \hat{t}_j)^2}{2(\delta\hat{t}_j)^2} \right\} \times \exp \left\{ \frac{-(\hat{T}_k(\eta_j) - \hat{T}_j)^2}{2(\delta\hat{T}_j)^2} \right\}. \quad (5)$$

over a uniform grid in $\hat{t} \in [0, 1]$. In this context, one can see the purpose for the term under the square root sign: in regions where the cooling curve is nearly vertical, the data covers fewer grid points than in regions where the curve is nearly horizontal. The term under the square root compensates for this, ensuring portions of the cooling curve which are nearly vertical get extra weight. This reweighting is relatively weak in comparison to the data, which exponentially affects the likelihood. We choose a grid of size 100 but increasing the number of grid points will not affect our basic conclusions.

The Markov chain Monte Carlo begins with an initial guess for the six superfluid parameters ($T_{c,peak,n}$, $k_{F,peak,n}$, $\Delta k_{F,n}$, $T_{c,peak,p}$, $k_{F,peak,p}$, $\Delta k_{F,p}$) and the envelope composition parameters. A new set of gaps and envelope compositions is randomly selected and the new likelihood is computed. The step is rejected or accepted according to the Metropolis algorithm. The autocorrelation length of all of the parameters is computed and the data is thinned in order to ensure the uncertainties in the parameters are properly estimated.

Quantity	Value and 1- σ uncertainty		
	w/o Vela or carbon	w/o carbon	all
$\log_{10} T_{c,\text{peak},n}$	8.133 ± 0.035	9.42 ± 0.14	9.307 ± 0.036
$k_{F,\text{peak},n} \text{ (fm}^{-1}\text{)}$	1.86 ± 0.17	2.040 ± 0.095	1.993 ± 0.014
$\Delta k_{F,n} \text{ (fm}^{-1}\text{)}$	0.45 ± 0.20	2.136 ± 0.080	2.057 ± 0.014
$\log_{10} T_{c,\text{peak},p}$	8.85 ± 0.63	9.60 ± 0.32	9.905 ± 0.066
$k_{F,\text{peak},p} \text{ (fm}^{-1}\text{)}$	0.593 ± 0.088	0.831 ± 0.057	0.934 ± 0.011
$\Delta k_{F,p} \text{ (fm}^{-1}\text{)}$	0.30 ± 0.14	0.116 ± 0.038	0.1076 ± 0.0069
$\log_{10} \eta_{\text{Cas A}}$			-10.70 ± 0.84
$\log_{10} \eta_{\text{XMMU J1732}}$			-7.61 ± 0.47
$\log_{10} \eta_{\text{PSR J1119}}$	-15.63 ± 0.95	-16.09 ± 0.57	-16.08 ± 0.13
$\log_{10} \eta_{\text{RXJ 0822}}$	-9.22 ± 0.87	-9.83 ± 0.50	-9.99 ± 0.13
$\log_{10} \eta_{\text{1E 1207}}$	-11.7 ± 1.1	-11.50 ± 0.32	-11.22 ± 0.12
$\log_{10} \eta_{\text{PSR J1357}}$	-8.80 ± 0.93	-9.11 ± 0.40	-8.95 ± 0.12
$\log_{10} \eta_{\text{RX J0002}}$	-16.77 ± 0.82	-16.77 ± 0.49	-17.49 ± 0.12
$\log_{10} \eta_{\text{PSR B0833}}$		-7.55 ± 0.37	-7.81 ± 0.11
$\log_{10} \eta_{\text{PSR B1706}}$	-8.17 ± 0.61	-8.39 ± 0.50	-8.56 ± 0.13
$\log_{10} \eta_{\text{PSR J0538}}$	-16.93 ± 0.84	-17.32 ± 0.35	-17.03 ± 0.10
$\log_{10} \eta_{\text{PSR B2334}}$	-8.08 ± 0.70	-7.32 ± 0.22	-7.32 ± 0.11

TABLE II: Posterior parameter values for three fits of the minimal cooling model to data. The first column labels the parameter, the second column gives results obtained without including Vela (B0833–45) or the carbon atmosphere stars, the third column includes Vela, and the fourth column includes all of the stars in the data set. The gap parameters depend most strongly on whether or not Vela is included in the fit. The envelope compositions are relatively insensitive to the data selection, but vary strongly between individual neutron stars.

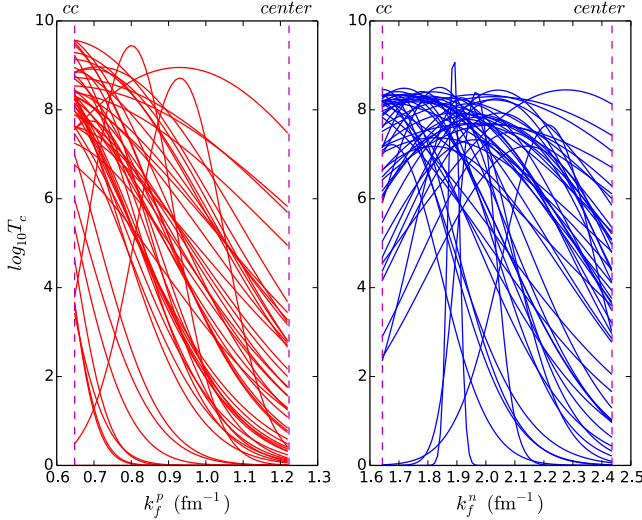


FIG. 2: Uncorrelated samples from the critical temperatures as a function of the Fermi momenta for protons (left panel) and neutrons (right panel) without Vela or the carbon stars. The left boundary in both panels represents the Fermi momentum at the crust-core transition (denoted “cc”). The right boundary represents the Fermi momentum in the center (denoted “center”) of a $1.4 M_{\odot}$ neutron star. Similar to previous works, we find stronger proton superconductivity and weaker neutron triplet superfluidity. The proton critical temperature most likely peaks at smaller densities and vanishes near the core while the behavior of the neutron critical temperature is less constrained.

III. RESULTS

We begin by removing the two carbon atmosphere stars in Cas A and HESS J1731–347 and Vela (PSR B0833–45). We perform a MCMC simulation as described above, assuming that the minimal cooling model holds, i.e. that the direct Urca process does not operate and that no exotic matter is present. The resulting gap parameters, the envelope compositions, and their uncertainties (which we have assumed symmetric about the central values) are given in the second column of Table II. The posterior cooling curves for $\eta = 0, 10^{-12}$ and 10^{-7} are plotted in Fig. 1 and a set of representative plots of the critical temperatures as a function of the Fermi momenta is given in Fig. 2. The posterior values for the peak critical temperatures are 7.1×10^8 K for singlet protons with a 20 percent uncertainty and 1.4×10^8 K for triplet neutrons with 10 percent uncertainty. The most probable values are in agreement with previous results from Ref. [54]. The cooling curves are displayed in Fig. 1. The early portion of the cooling curves are only weakly dependent on the gaps before a large portion of the neutron star core has undergone the superfluid/superconducting phase transition. The representative samples of the critical temperature curves in Fig. 2 show that the proton superconducting gap is likely largest just near the crust core transition and falls off dramatically at the highest densities in the core. The triplet neutron superfluid critical temperature, on the other hand, may peak at any

density so long as a large enough portion of the core undergoes the superfluid phase transition.

The quantitative nature of our fit also allows us to determine the envelope composition for H atmosphere neutron stars. We find PSR J1119-6127, RX J0002+6246 and PSR J0538+2817 all most likely have no light elements in their envelopes, in contrast with a small amount of light elements in 1E 1207.4-5209 and a significant contribution from light elements in all of the other H atmosphere stars. Note that stars which lie to the left and below the cooling curves tend to have a large amount of light elements, choosing to fit more to the $\eta = 10^{-7}$ curve lying to the right of the data point than attempting to fit to the $\eta = 10^{-17}$ curve above the data point (because the time uncertainty is larger than the temperature uncertainty).

Now we add Vela and redo the fit. The results are summarized in the third column of Table II, and Figures 3 and 4. This one data point, lying to the left and below the curves, has a strong impact: the critical temperatures implied by the data are much larger than that obtained previously. This increases the width of the cooling curves at early times in Fig. 3 because the superfluid/superconducting phase transition happens early in the thermal evolution. Vela is an outlier, and it is well known that outliers can dramatically affect the mean. This is a result of the age revision of Vela down to $5 - 16 \times 10^3$ years as obtained in Ref. [20] and discussed in Ref. [54]. The envelope compositions are unchanged (within errors) and the fit prefers a significant amount of light elements in Vela's envelope in order to become closer to the $\eta = 10^{-7}$ curve lying to the right. Fig. 4 shows that the proton superconductivity is slightly stronger (15%) and driven to a higher density (in order to allow a larger part of the star to participate in the Cooper-pair neutrino emissivity). The neutron superfluid gap is broader, requiring almost the entire star to participate in neutron triplet superfluidity.

While the absolute normalization of the likelihood function is not meaningful, relative values are physical. A typical data point contributes a factor of 0.5 to the likelihood while Vela's contribution is 10^{-3} . This is a strong indication that fitting Vela is difficult in the minimal cooling model. The observation of Vela, as it currently stands, provides some evidence for the direct Urca process or the presence of exotic matter in neutron star cores.

Previous works [55, 56] found very strong constraints on proton singlet superfluidity and neutron triplet superfluidity from observations which implied the neutron star Cas A had cooled over a 10-year period of observations [11]. Refs. [57, 58] find a different result, demonstrating agreement with the Cas A neutron star required in-medium effects on the thermal conductivity and a particular proton gap and was insensitive to the neutron triplet superfluidity. Ref. [59] finds similar constraints on the gaps as Refs. [55, 56], and employs a polynomial parameterization of the gaps (in contrast to the Gaus-

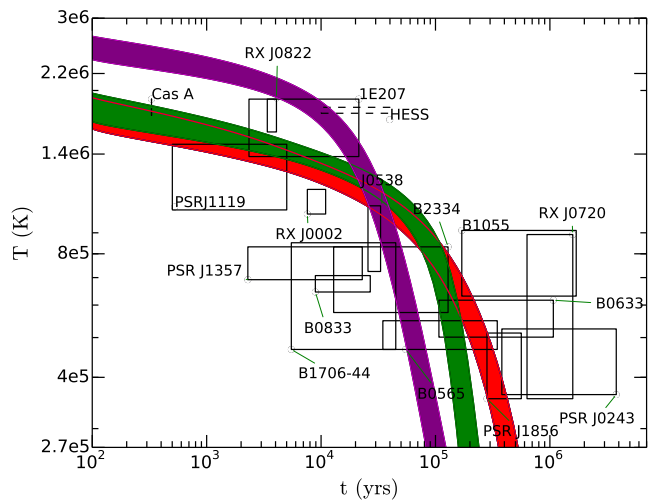


FIG. 3: Cooling curves plotted as in Fig. 1, but now having added Vela (B0833-45) to the analysis. The fit implies stronger neutron superfluidity and stronger proton superconductivity. This makes the cooling curves are wider at early times and drop to the left at late times in order to fit the Vela data point. These results correspond to the parameter limits in the third column of Table II.

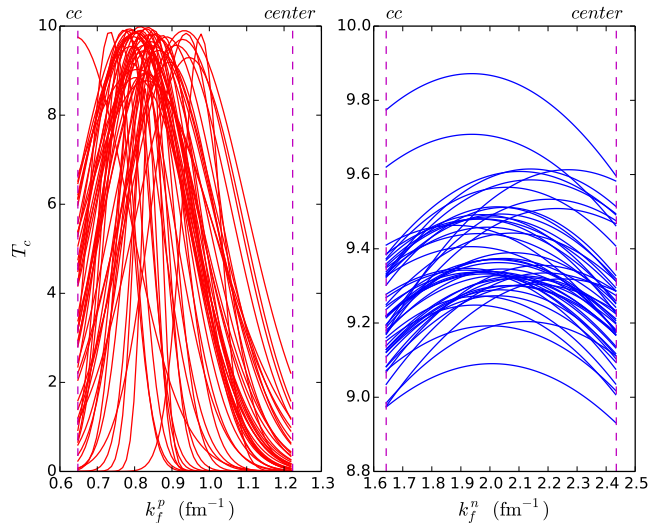


FIG. 4: Uncorrelated samples from the the critical temperatures as in Fig. 2 now having added Vela (B0833-45) to the analysis. The typical value of $T_{c,\text{peak}}$ for the protons is higher, and the typical neutron triplet critical temperature curve is much broader.

sian form we use in Eq. 1). Recent observations of the neutron star in Cas A imply that it may not have cooled appreciably in the past 15 years [60, 61]. For this work, we assume that the systematics do not enable us to constrain the cooling over a short timescale.

In this case, adding the neutron star in Cas A to the data set does not make a strong modification in our results. The surface temperature of Cas A lies in between

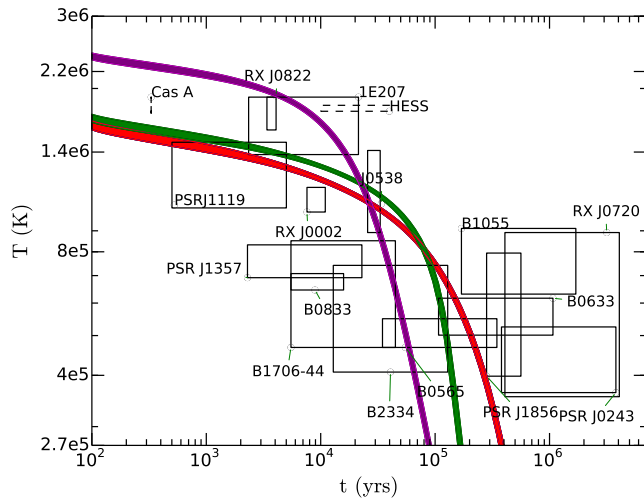


FIG. 5: Cooling curves plotted as in Fig. 1, but now having added Vela and the carbon-atmosphere neutron stars to the fit. The superfluid/superconducting critical temperatures are very well determined because of the tension between Vela and XMMU J1732. These results correspond to the parameter limits in the fourth column of Table II.

the results for envelopes with and without light elements so our result simply chooses a moderate amount of light elements $\eta \sim 10^{-10}$ in order to explain the data. However, adding the other neutron star thought to have a carbon atmosphere, XMMU J1732, creates a strong preference for warmer stars with light element envelopes. This is in strong tension with Vela, which has a strong preference for cooler stars with light element envelopes. This tension results in very tight constraints on the superfluid properties of dense matter, as summarized in the fourth column of Table II, Fig. 5 and Fig. 6. In the context of Bayesian inference where the evidence for a particular model is determined by the integral over the likelihood, the dramatic decrease in the parameter uncertainties leads to a model with very small evidence. In other words, if Vela and XMMU J1732 are confirmed to have ages and temperatures near the central values reported in Table I, then it is likely that a model with some additional parameter which enables faster cooling in Vela will provide a much better fit.

IV. DISCUSSION

Most importantly, our work *quantifies* the extent to which superfluid properties can be constrained from currently available data on the cooling of isolated neutron stars. Most of the previous works on this topic give more qualitative results: they do not employ any particular likelihood function and thus cannot give full posteriors for their parameter values. The extent to which our quantitative approach will be possible without making the assumptions of the minimal cooling model will be

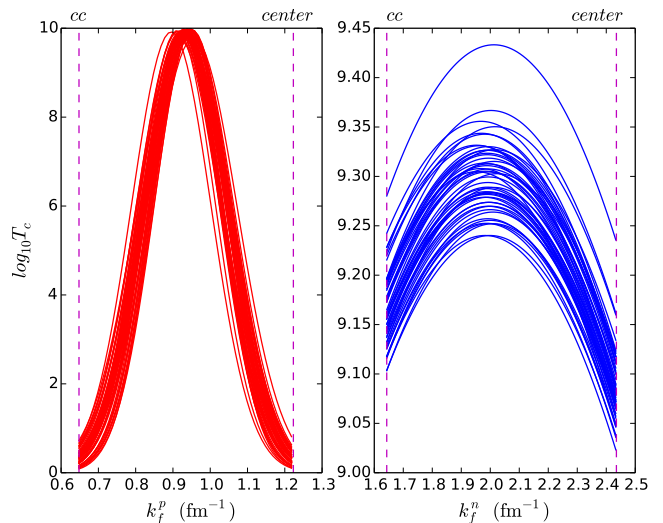


FIG. 6: Uncorrelated samples from the the critical temperatures as in Fig. 2 now having added Vela and the carbon-atmosphere neutron stars to the fit.

explored in future work.

Our analysis has between 14, 15, or 17 parameters corresponding to 14, 15, or 17 data points, respectively. (The fact that the number of parameters is equal to the number of data points is coincidental: we have six superfluid parameters and six black-body atmosphere neutron stars in our data set). One of the advantages of our Bayesian approach is that our formalism does not require the fitting problem to be over-constrained. Had we not employed the minimal cooling model, we would have required at least 4 new parameters to describe the EOS and an additional mass parameter for each neutron star (bringing us to a total of 38 parameters for 17 data points). An accurate mass measurement for even a few of the neutron stars in this data set would improve the fitting problem substantially.

One possible extension would be to attempt to explain the surface temperatures of accreting neutron stars as well, as done in Refs. [62] and [63]. It is well known that some of those objects, in particular SAX J1808.4–3658, are too cold to be explained within the minimal cooling model [64], and thus the direct Urca process is invoked. The approach taken in Refs. [62] and [63] is similar in that they employ a systematic exploration of their parameter space, it is different in that they do not explicitly compute the likelihood of their models as we have done in Eq. 4. Extending our method to include the direct Urca process would necessitate also considering the variation in the EOS as well.

Several authors have examined the cooling of isolated neutron stars outside the minimal model. Ref. [20] examined cooling with hyperons, and finds that superfluidity is required to ensure that the direct Urca process does not make neutron stars too cold. Refs. [65–68] obtain a strong EOS dependence in their results due to the fact

that they allow the direct Urca process. They, along with Refs. [62, 63], find that the data can be explained without exotic matter so long as the direct Urca process operates in some stars. We find (as first found in Ref. [7]), that the isolated neutron stars (with the exception of the Vela pulsar) can be easily explained without having to invoke the direct Urca process, so long as one allows for variations in the envelope composition at early times. Ref. [69] has invoked axions in a model which does not include the direct Urca process. While we are performing our work in a model which contains more restrictive assumptions about the nature of dense matter, our statistical analysis allows us to be more quantitative in our conclusions. Extensions of this work beyond the minimal cooling model are in progress.

For the neutron stars with a carbon atmosphere, Ref. [59] performs a χ^2 fit to the data for the neutron star in Cas A, under the alternative assumption that this neutron star is indeed cooling quickly as found in Ref. [11]. A χ^2 fit is possible here because there is no uncertainty in the x-axis, and thus the likelihood function in Eq. 4 gives the same result. We include a larger data set and perform our Monte Carlo over a much larger set

of cooling models. Ref. [70] also assumes that Cas A is cooling quickly, and explains the data using a neutrino emissivity from superconducting quarks. The cooling of the carbon atmosphere star XMMU J1732 has been addressed in Ref. [71] who also found a large heat blanketing envelope was required to reproduce the data. They obtain a constraint on the mass and radius of this neutron star because, in their model, the proton superfluid gap is correlated with the mass and radius. In contrast, we treat the EOS and superfluid properties of matter as independent.

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